TO MOVE THROUGH SPACE:
lines of vision and movement

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Abstract

Space syntax studies of pedestrian behaviour in building and urban environments have shown that there is a consistent correspondence between the configuration of space and the patterns of usage found within it. In particular, it has been shown that the topological relationships within a spatial system correlate to observed aggregate pedestrian movement. However, there is no proposed mechanism supporting the theory at the level of the individual. Although links between space syntax and individual movement decisions have been suggested through way-finding studies of building environments, virtual reality experiments, and agent-based models, none have proposed a formal link to the axial line analyses used within space syntax.

Here we extend work on agent-based models to build a bridge between the line-based topological analyses of space syntax and visually directed agents, through the analysis of what we call 'through vision'. The decision rules for visually directed agents form a Markov transition matrix. We recap the mathematics of Markov chains in order to show that the steady state movement corresponds to an eigenvector of the transition matrix. As the agent transition matrix is extremely complex, we demonstrate that a good approximation of the eigenvector is achieved through the summation of the lines of vision through any one location within an environment. This set of lines forms a superset of the all-line axial map comprising the edges of the visibility graph, or lines of through movement. We show that the lines may be reduced in number (or bundled together) by an algorithmic process and connected into paths, thus making a direct connection between a moving individual with vision and the space syntactic topological analysis of space.

Introduction

Although this paper is quite mathematical in nature, the motivation for it stems from phenomenological concerns. At the heart of any space syntax analysis is a representation of a system in an abstract form, be it a set of convex spaces or a set of axial lines. There is an action of reduction from the 'living' world to a series of lines inscribed on a map. This is not to say that an inscribed line cannot itself be 'living'. Ingold (2005) examines the life of the line. To Ingold, a freely drawn line is alive. He discusses the importance of the distinction between a 'trail'

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and a 'route'. A trail is the path taken by the inhabitant of a space, and thus alive, whereas a route is prescribed, and tends towards transport, where the traveller is no longer active in their choice of direction. For a line to be living, therefore, it must pertain to the individual's self-motivated path through the environment. It is clear that, for an axial line to be 'alive', it must represent not an a priori route through the environment, but an embedded set of trails of individuals. Although to Hillier and others, the axial line is undoubtedly an approximation of this living entity, the definition of the axial line is itself a mathematical artefact (Turner et al 2005). The resolution in this paper is still founded on algorithmic implementation; however, it will form an attempt at mathematical retrieval, rather than mathematical construction, of the axial line.

In order to retrieve axial lines, we will need to begin with the individual within the environment, and in doing so, we will have to examine the modes of interaction available to the individual. Phenomenologists such as Seamon suggest that the modes of interaction, or the ontological basis for our understanding, should be arrived at through introspection of what there is (Seamon, 2003). It seems to me that the definition of 'what there is' is made more accessible through the theory of autopoeisis (Maturana and Varela, 1980). In autopoeisis, the individual is a self-sustaining entity within the environment. The observer is given no access to the internal structure of the agent, and so may only look to the relationships between the individual and the environment in order to understand the possible modes of engagement of the two.

The visual relationship is one that seems to be apparent. Ingold himself makes the connection with Gibson's theory of natural vision, suggesting that we perceive the world along a 'path of observation':

> Proceeding on our way things fall into and out of sight, as new vistas open up and others are closed off. By way of these modulations in the array of reflected light reaching the eyes, the structure of our environment is progressively disclosed (Ingold 2005: 49).

It is important that the disclosure is through a set of straight lines, those of Gibson's ambient optic array. This is a physical constant to us, but there are other physical constants that act in straight lines. The process of bipedal movement tends to a straight line (albeit less so that quadrupedal). Furthermore, we must consider the fact that the development of the brain and its capability of understanding must be governed by the geometry of the environment (O'Keefe, 1993). If this is true, then we would expect navigation strategies in the visually impaired to follow those of the able-sighted, for which there is evidence (Golledge et al, 1996).

Thus, for the purposes of this paper, there is an ontological decision that there are straight-line connections, and there are bodies capable of acting according to these connections.

It is not just for philosophical reasons that a connection must be made between the individual in the environment and axial lines. From a pragmatic point of view, there have now been many studies which are considered to be 'space syntactic', in that they consider the relationship of spaces to other spaces, but relate far more to the individual within the space (or the potential of the individual to occupy a space). These include Peponis et al's (1990) and Haq's (2003) studies of individual wayfinding activity, Conroy Dalton's (2003) experiments on navigation within virtual reality environments, and Turner and Penn's (2002) agent-based models of individual movement patterns, as well as Turner et al's (2001) visibility graph analysis. Each has made a finding of the importance of straight lines and a claim that there is connection between their findings and the entities of space syntax, in particular, the axial line. However, despite the fact that there is a clear correspondence in methodological approach, there is no formal link between the individual and axial lines. For space syntax to be complete, this link must be made.

The paper comprises three major further sections. The next section, *Agents and Eigenvectors*, will start with the agents presented in Turner and Penn (2002), and guide through the mathematics to show that the formal link to axial lines seems to lie with the set of all possible straight lines that can be drawn between open spaces in a system. The section will guide the reader through the mathematics of the paper in three stages:

1. There is an analytic result that is the same as running the agents. That is, by considering how the agents move, it is possible to predict the outcome without running the agents.

2. The general analytic result is too complex to calculate for the set of all parameter inputs to the agents. There is, however, a specific analytic result for a subset of the agents

3. The specific analytic result is simply the summation of all possible sight-lines through a location.

The *Through Vision* section discusses the practical implementation of the method. We will have to approximate, and introduce a Cartesian grid, of which Ingold would disapprove. However, the methodology remains distinctly physical: it is an experiment in the pattern of behaviours that are possible given physical constraints of systems. The section shows application to abstract examples, as well as benchmark tests on the Tate Britain Gallery, London, and the area of Barnsbury, North London, and shows that the theoretic analytic result is strongly correlated with the result of running the agents.

The *Discussion* section examines how the set of through vision lines may be bundled up into a set of fewer axial lines. It is shown that the same subset reduction algorithm presented in Turner et al (2005) may be applied to through vision lines to produce an axial map, while philosophically it is considered that the algorithm might better be called a "superimposition". That is a superimposition of the possible trails through the environment, or the collection of Ingold's living lines.

Agents and Eigenvectors

An agent, or animat, is a computer simulated actor within an environment, which is guided by simple rules. An exosomatic visual architecture (EVA) agent is guided by vision of the environment through sampling the available open space location at its current location (Penn and Turner, 2002). It selects a destination from a field of view, and takes several steps towards its destination before selecting another destination. In this way it progresses through the environment as shown in figure 1.

It was demonstrated that EVA agents correlate extremely well with pedestrian movement in the Tate Britain Gallery, London, with a correlation coefficient of up to R^2=0.76 (Turner and Penn 2002). Although slightly less compelling, EVA agents have also been shown to correlate well with pedestrian movement in urban environments, with a correlation coefficient of up to R^2=0.67 (Turner 2003).

It might seem that, as the agents select destinations at random, the result of releasing a set of agents into the environment will differ. However, the results are in fact extremely similar, due to the fact that the agents form a Markov chain (Turner and Penn 2002). A Markov chain is simply a chain of events where the next event does not depend on the previous event (see Aldous and Fill 2002 for detailed explanation of the mathematics presented herein). In agent terms, the agent's choice of destination at one location does not depend on its choice of destination at the last location it visited. A Markov chain is characterised by a *transition matrix*. The transition matrix gives probabilities of moving from one location to another. In order to grasp the function of a transition matrix, it is probably best to think of a simple example. Think of four towns joined together by roads as shown in figure 2.

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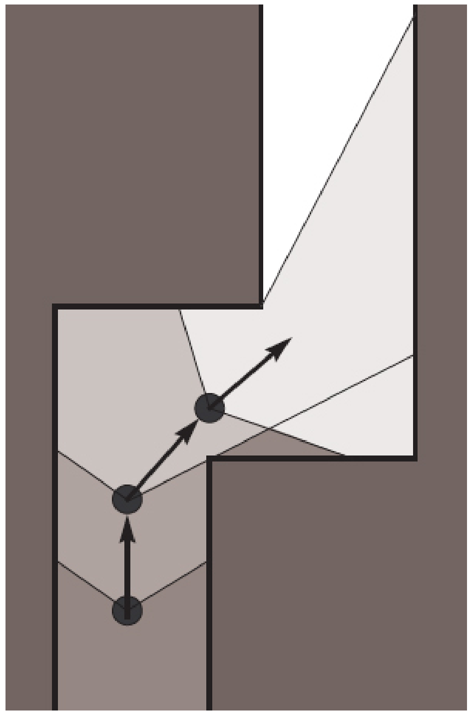

Figure 1:

An exosomatic visual architecture (EVA) agent tends to move onwards through an environment due to repeated reselection from a field of view (far right)

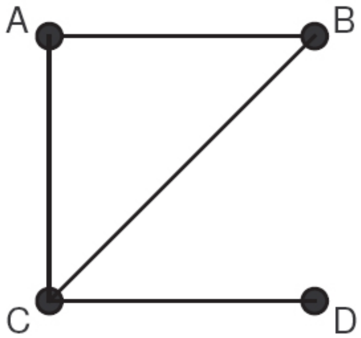

Figure 2:

A system of four towns joined by four roads

If 24 people were to start out from town A, and choose a destination at random, they could choose to move to either town B or town C, but not town D. Since the destination is random, they would choose town B and town C with equal probability. On average ½ would go to town B, and ½ to town C, or 6 to each. Similarly, from B ½ would go to A, and ½ to C. However, from C there are three connections, so 1/3 each would go to A, B or D. Finally, from D, assuming everyone moves every step, all must return to C. These transition probabilities are written as a matrix as shown in equation (1).

$$\begin{array}{cc} \begin{array}{cccc} A & B & C & D \end{array} & \\ \begin{pmatrix} 0 & \tfrac{1}{2} & \tfrac{1}{3} & 0 \\ \tfrac{1}{2} & 0 & \tfrac{1}{3} & 0 \\ \tfrac{1}{2} & \tfrac{1}{2} & 0 & 1 \\ 0 & 0 & \tfrac{1}{3} & 0 \end{pmatrix} & \begin{array}{c} A \\ B \\ C \\ D \end{array} \end{array} \quad (1)$$

The columns in the matrix shown in equation (1) give the probabilities of moving to each of the other locations. So from A, there is a probability of ½ of moving to B and a probability of ½ of moving to B. We can show what happens to 24 people by multiplying the matrix by a vector representing the location of the people, as shown in equation (2). 24 people start at town A, and 12 end up in B and C respectively:

$$\begin{pmatrix} 0 & 0.5 & 0.333 & 0 \\ 0.5 & 0 & 0.333 & 0 \\ 0.5 & 0.5 & 0 & 1 \\ 0 & 0 & 0.333 & 0 \end{pmatrix} \begin{pmatrix} 24 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 12 \\ 12 \\ 0 \end{pmatrix} \qquad (2)$$

The situation after the next move can be shown by multiplying the resultant vector by the transition matrix once more, as shown in equation (3):

$$\begin{pmatrix} 0 & 0.5 & 0.333 & 0 \\ 0.5 & 0 & 0.333 & 0 \\ 0.5 & 0.5 & 0 & 1 \\ 0 & 0 & 0.333 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 12 \\ 12 \\ 0 \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \\ 6 \\ 4 \end{pmatrix} \qquad (3)$$

Note that the sum total of people remains at 24, 10 now at A, 4 at B, 6 at C and 4 at D. Now a question naturally arises: how many people would be located in each town if the system were left to run for an infinite amount of time? One important property of a transition matrix such as this one is that the number converges to a *stationary distribution*. That is, after an infinite number of steps, the number of people at any one location is entirely predictable.

At this point it is worth noting that we have set up a graph, with each town as a node and each road as an edge. For a transition matrix such as this, where it is equally probable that one makes a journey to any other node, the matrix is said to be *time reversible*. For a time reversible matrix, we can calculate the stationary distribution by counting the out edges from each node, and dividing by the number of edges multiplied by two (essentially, one edge out, one edge in makes double the number of edges). So, for A there are two edges leading out, and there are 4 edges total, so 2/8, or 6 of 24 people will end up at A. For B it is also 6, for C it is 3/8 of 24 or 9 and for D 1/8 of the 24 or 3. We can check this really is the stationary distribution by multiplying by the transition matrix:

$$\begin{pmatrix} 0 & 0.5 & 0.333 & 0 \\ 0.5 & 0 & 0.333 & 0 \\ 0.5 & 0.5 & 0 & 1 \\ 0 & 0 & 0.333 & 0 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \\ 9 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \\ 9 \\ 3 \end{pmatrix} \qquad (4)$$

Note, that for a time reversible Markov chain, this simple relationship between adjacency and stationary distribution always holds (Wagner 2003). The property has significant implications for axial as well as agent systems: if people were moving randomly over an axial system for infinite steps, they would end up in proportion to the connectivity of each axial line. It is useful to note that connectivity is usually a good correlate with pedestrian or traffic movement, and thus this approximation (infinite movement) is a useful one.

In more formal terms, the vector for the stationary distribution is called an eigenvector [i] of the transition matrix. The eigenvector has the property that $M\boldsymbol{\omega}=\lambda\boldsymbol{\omega}$, where M is the transition matrix, $\boldsymbol{\omega}$ the eigenvector, and λ is an eigenvalue. In this case $\lambda=1$. The property of convergence to the stationary distribution suggests that any startingvector will end up as $\boldsymbol{\omega}$ if multiplied by M enough. That is, for an arbitrary vector $\mathbf{x}$:

$$\lim_{n \to \infty} M^{n}\mathbf{x} = \boldsymbol{\omega} \tag{5}$$

This equates essentially to: run the agents for long enough from any starting position, and the stationary distribution will be achieved.

Now, for time reversible systems, the eigenvector is easy to calculate. Unfortunately, the standard agents with forward facing vision are not time reversible. That is, if an agent moves from location A to location B, then it ends up facing the same direction it started. Thus, having moved from A to B, it cannot reverse and return to A without first turning around, which it must achieve through further steps through the transition matrix. Worse still, it is, in general, extremely complex to calculate the eigenvector of a matrix. It is probably preferable to run the agents for an extended amount of time instead [ii]. However, there is a form of agent that is time reversible: if the agent can see in all directions, then it can return to the location it just left. Hence, the eigenvector for an agent with 360° field of view can be calculated efficiently. It is simply proportional to the number of out connections, or visibility connections, from that location. However, we are not directly interested in the number of agents that end up at any one location, but the *movement* from one location to another. If we further posit that, in addition to the 360° field of view, an agent continues to its destination, then it is clear that any pair of intervisible connections have an equal chance of being chosen as origin and destination in the stationary distribution. Therefore, there is a single journey for each origin destination pair of sight lines in the system, and as the agent movement is proportional to the number of journey crossing through the location, the agent movement of through a location is equal to the number of sight lines crossing it [iii].

Of course, in producing this approximation, we have given up two major facets of the standard agents: the forward facing movement, and the continuous reselection of destination. However, in open systems, such as the Tate Britain Gallery, the 170° field of view agent can easily return on itself, almost completely in two steps by turning first 170°, taking three steps and then taking a second turn of 170°. Hence, the visual field acts as a 360° agent with a longer distance between steps. In urban systems, where the streets are narrower, the effect is more limited as the probability of turning to side is significantly lower, but, as we shall find, this may not be as useful as it appears.

Through Vision

In this section through vision will be defined formally, and it will be demonstrated that empirically it does correspond to agents released with a 360° field of view which continue to their destination. Furthermore, we will show that for the Tate Britain Gallery, the through vision is a good approximation of standard 170° field of view agents taking 3 steps. We will also show that the through vision also corresponds well to actual pedestrian movement in the Barnsbury area, suggesting that through vision may in fact lead to an improvement of our understanding of how to program agent rules for urban systems.

Through vision is defined for a dense grid visibility graph (Turner et al 2001). The visibility graph is calculated for each point on a dense grid. Each edge in the visibility graph (the intervisibility line between two locations) is pixelated at the resolution of the grid. Then, each point on the pixelated line is incremented by a value of one for each line that passes through it. The lines contributing to a highlighted location are shown in figure 3. The value of through vision for this point is 56. In mathematical terms:

$$T(x) = \sum \gamma(a, x, b) \quad \forall a, b : e_{ab} \in E \qquad (6)$$

where γ(a,x,b) = 1 if the pixelated line between a and b passes through x and 0 otherwise. e_{ab} is an edge joining a and b, and E the set of all edges in the visibility graph.

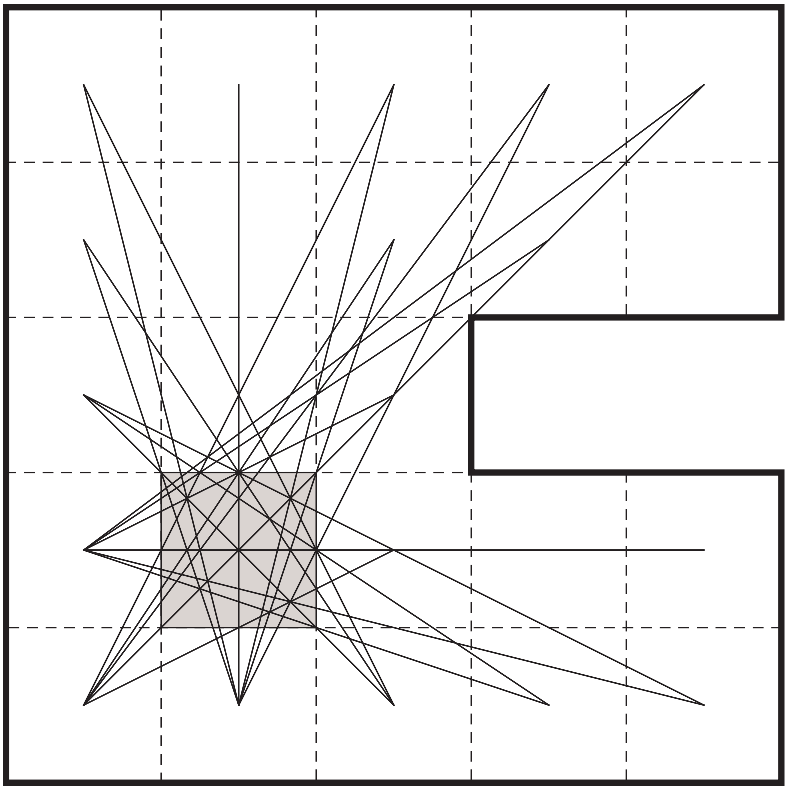

Figure 3:

The lines contributing to the through vision value for a location

The through vision was calculated for four systems: a simple square, an abstract shape, a plan of the Tate Britain Gallery London, and the area of Barnsbury in North London. In each case the values obtained were compared against standard 170° field of view agents which take 3 steps between movement decisions (labelled 15/3), and 360° field of view agents which continue to their destination (labelled 32/inf). The agents were run with a minimum number of agents with each agent taking 1,000,000 steps through the environment in order to obtain a result as close as possible to the stationary distribution. It should be noted that all these experiments are theoretical in nature, and simply intended to show that there is a relationship between a theoretical and empirical stationary distribution; the experiments reported in Turner and Penn (2002) and Turner (2003) use agents which take far fewer steps in order better approximate actual pedestrian movement.

Figures 4, 5, 6 and 7 show the results of comparison between through vision and agents for each of the square, abstract shape, Tate and Barnsbury respectively. In each case, the scatter between the agent values (x axis) and through vision values (y axis) are shown. As can be seen, for the simple shapes, and Tate Gallery, there is good correspondence between both types of agent and through vision. Just as was anticipated in the last section, the open areas of the Tate Gallery do mean that 3-step 170° field of view agents are compatible with destination-step 360° field of view agents. Conversely, the system does break down in the case of Barnsbury. Although the through vision still matches the 360° field of view agents as the theory predicts, there is a significantly lower correlation between 3-step 170° field of view agents and through vision. However, there is a remarkable result if, rather than comparing the values with each other

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on the sample systems shown, we compare with gate counts of pedestrian movement.

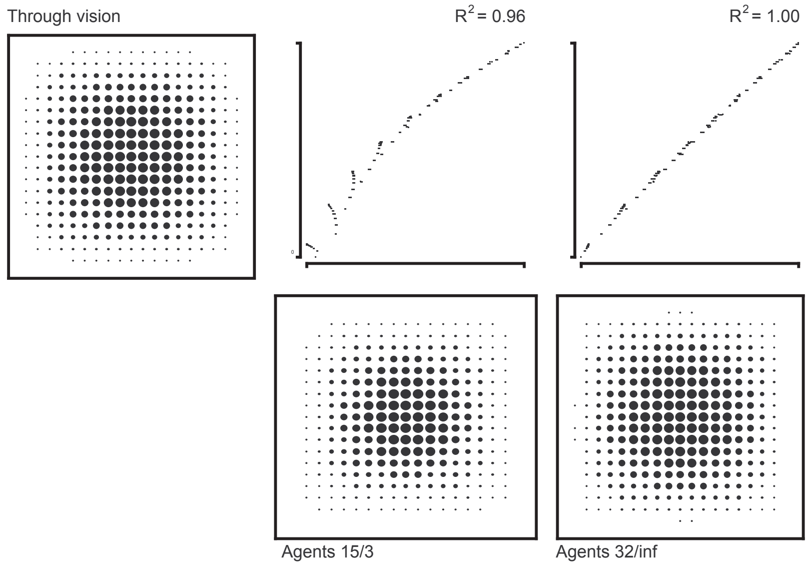

Figure 4:

Comparison of through vision with agent experiments for a square shape

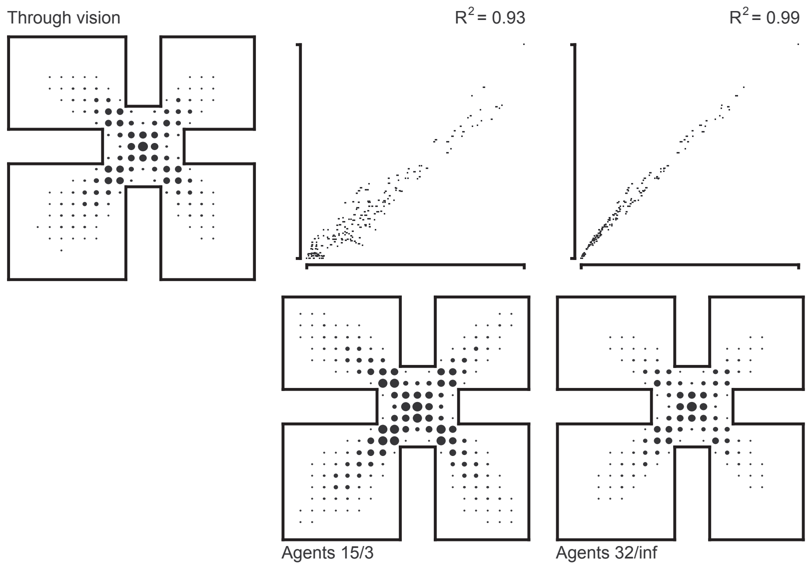

Figure 5:

Comparison of through vision with agent experiments for an abstract shape

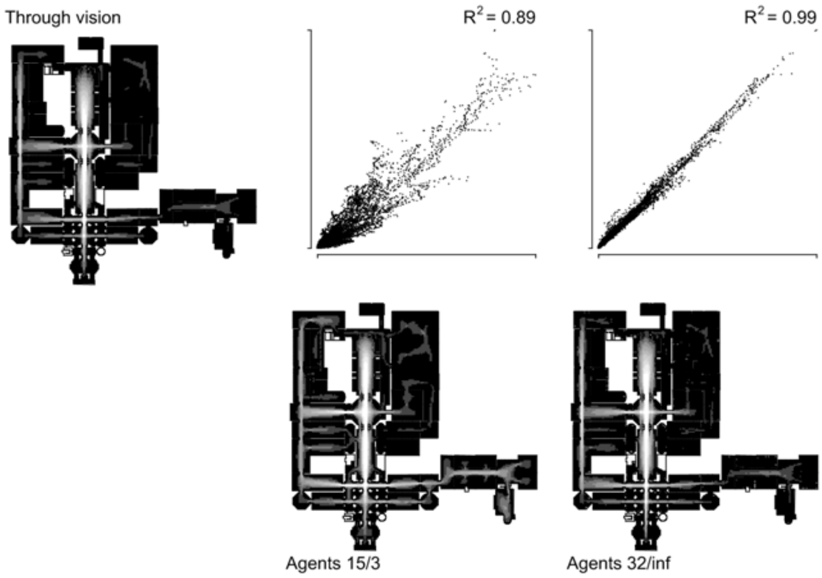

Figure 6:

Comparison of through vision with agent experiments for the Tate Britain Gallery, London

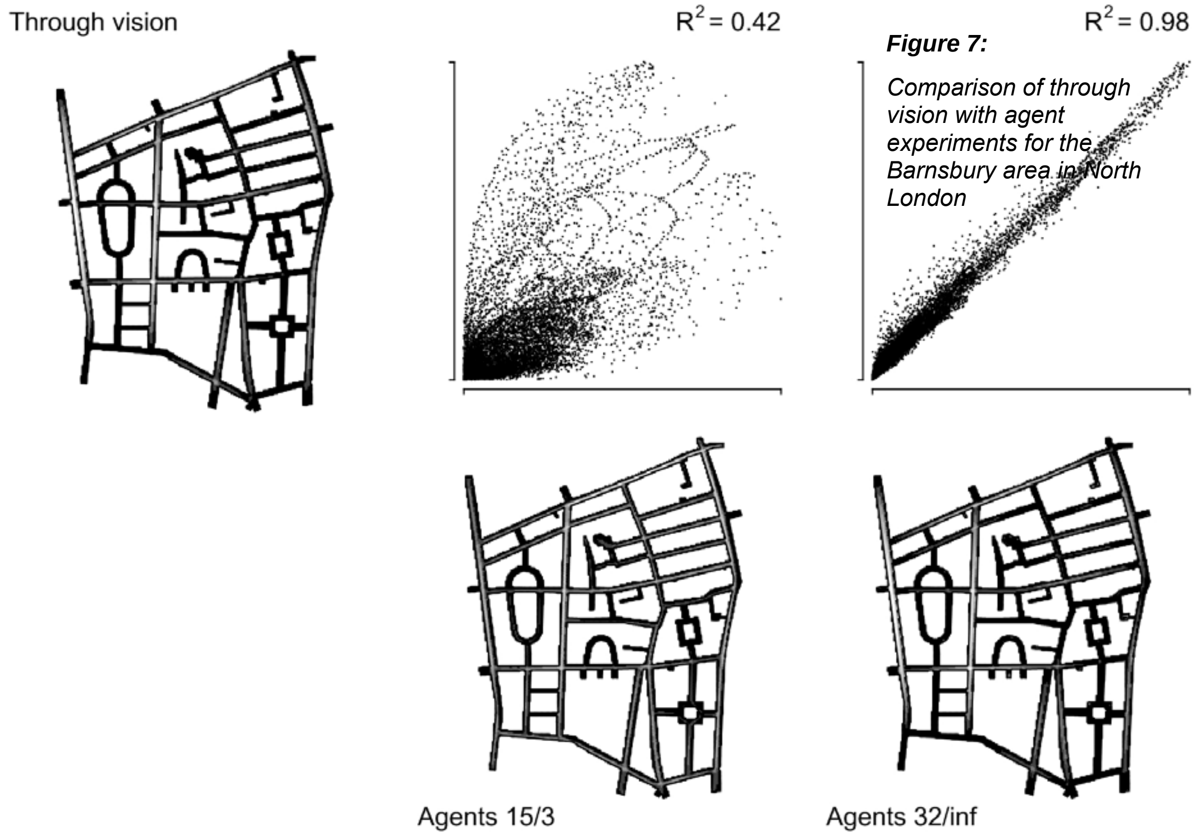

Figure 7:

Comparison of through vision with agent experiments for the Barnsbury area in North London

The agent and through vision experiments were applied to a larger model of the Barnsbury area, taking into account the surrounding area, and compared with 116 gate counts of pedestrian movement reported by Penn and Dalton (1994), taken around the central region shown in figure 7. The correlation with pedestrian movement is contrary to what might be expected: the 3-step 170° field of view agents correlate with R^2=0.46, while the through vision (and 360° field of view agents) with R^2=0.62. It should be borne in mind that these are results for the stationary distribution, as if the agents walked forever, rather than the shorter paths we might expect pedestrians to take; equally, though, the difference is significant enough to suggest that the application of 3-step 170° field of view agents might be misguided, and that there should be further consideration of why the current agents perform better.

Given the result for the Barnsbury area, the same experiments were carried out on the Tate Gallery, comparing pedestrian movement through rooms recorded by Hillier et al (1996) with the through vision and agent experiments. It was found that the 3-step 170° field of view agents correlate with R^2=0.74, while the through vision (and 360° field of view agents) with R^2=0.68. Both these experiments are also somewhat remarkable in that they take no account of entrance to the gallery, which was used to release agents in the original EVA study (Turner and Penn, 2002). Again, it appears that the stationary distribution is a good indicator of movement levels, despite being a coarse approximation to the real world.

Discussion

Hillier (2001) deliberates on the effect of partitioning space, both from a metric and a visual point of view. He shows that the position of a block in one dimension has a square effect on the number of metric routes through a location. What he has actually arrived at though is the through vision in one dimension. Furthermore, Hillier remarks on the dual relationship between vision and physical metric contingency.

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Again, the systems which Hillier describes are analogous to those presented here. The line of visibility is combined with the physical path it described to create the measure of through vision. In doing so, the measure rescues the usefulness visibility graph analysis. Despite initial promise, visibility graph analysis did not appear to correlate to movement levels. In fact, the first results reported a correlation with occupation rather than through movement (Turner et al, 2001). It is now clear why that was the case: the visual connectivity is correlated with occupation in the Tate Britain Gallery because the eigenvector of the transition matrix is proportional to the visual connectivity, and the eigenvector represents the number of individuals at a certain location in the steady state. Of course, the correlation with movement could not be found, as the movement relies on the through movement between the spaces of occupation. As has been shown above, when this through movement is considered, the correlation with movement is found (at R^2=0.68). Indeed, we might think of this steady state distribution as the 'natural harmonic' of the building, allowing us to make the comparison between a functioning building such as the Tate Britain Gallery where the movement from the entrance naturally aligns with the steady state movement potential, and a non-functioning building where the release of agents from the entrance would not correlate with the steady state. Our assumption is that understanding of the building would be easier if the visual relationships within it match the entrance and function of the building, although this paper has steered somewhat clear of the cognitive implications of through vision. This is partly deliberate: the theory of through vision is entirely based on the circumstance of network relationships in a system, and does not provide for how the agent within the system makes use of them. However, recently there has also been an attempt among the space syntax community to relate cognition at the individual level with space syntax (Hoelscher et al 2007). Although this paper is not cognitive, it does trigger the possibility of a retrieval of a cognitive representation [iv].

At the start of the paper, we mentioned that there was a relationship of through vision with axial lines. It seems to me that the axial line turns towards a cognitive structure that might be retrievable from the lines of through vision. If we consider how the through vision lines interact, they build up along long lines rapidly, as there are many more pairs of visibility along a long line. Thus, we might consider that these may be bundled together into a single superline. In fact, this is much what the reduction methods for axial line map retrieval of Peponis et al (1998) and Turner et al (2005) do. The algorithm in Turner et al (2005) is specified as a subset reduction, which takes as its starting point an all-line axial map. However, the all-line axial map does not have to be the starting point: any set of lines may be used within the subset reduction. In fact, at high enough resolution, the through vision lines form a superset of the all-line map. The all-line map lines include the longest, and therefore most connected lines from the superset, and so any subset reduction will of the through vision lines will result in the same outcome as a subset reduction of the all-line map. Thus the through vision lines may be considered as a starting point to discover the most connected lines among them. From a cognitive point of view, this set will be an efficient representation of the possible movements through any space, and therefore, through Occam's razor, more probable than other cognitive mapping strategies.

Finally, the research leads to a couple of directions for further work: firstly, it appears the outcome should also bear a relationship to a Fourier transform of the space, as the superimposition of lines is also related to the superimposition of frequencies in the space; secondly, an approximate eigenvector for restricted field of view agents might easily be achievable by adding a backtrack possibility to the transition matrix. In general the backtrack step would be one out of many choices available to the agent, and so its effect on the agent movement pattern would be minimal. However, as the backtrack would make the transition matrix time reversible, it would allow an eigenvector to be retrieved from the connectivity as we have shown herein.

Conclusion

This paper introduced the concept of "through vision", the summation of all visibility lines for each location on a visibility graph grid. It was shown that the through vision is equivalent to the eigenvector of transition matrix for an agent with 360° field-of-view that continues to its destination. The importance of this result is that through vision yields the analytic result of an agent simulation run, and thus establishes connection between agents and lines through a system. The lines may be grouped together in order to create an axial line system. On a philosophical level, this means that a relationship between the traditional axial line analysis of space syntax may be brought together with an understanding of the individual within an environment. This serves two goals: for the phenomenologist, it establishes a link between life and line, for the scientist, a working framework to provide a mechanism for the working of an axial line. Furthermore, it correlates well with pedestrian behaviour in both an urban (R^2=0.62) and building scenario (R^2=0.68), implying that the relationship between life and the line may well be fundamental to how we move within the environment.

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i. Eigen-, from the German meaning 'same', hence an eigenvector remains the same when multiplied by a matrix.

ii. The calculation of an eigenvector for a non-time reversible transition matrix is beyond the scope required for this paper. However, it is relevant to current work on space syntax. One way to calculate the eigenvector is to note that the stationary distribution is related to the time it takes to return to each location. That is, the eigenvector is dependent on the access time, or the expected time to reach each vertex in the system. The access time can be calculated recursively, by summing the probability of getting from one vertex to any other:

$$H(a,b) = 1 + \sum_{x} p_{ax} H(x,b) \qquad e_{ax} \in E$$

$$H(b,b) = 0$$

where H(a,b) is the access time from a to b, pax is the transition probability from a to x. eax is an edge between a and x, and E the set of all edges, hence the sum at each step is for all the locations connected to a. The recursion ends when we reach the destination b. This equation is of general interest to space syntax not as a method of calculating the eigenvector, since as discussed in the body of the paper, an axial system is time reversible and thus the eigenvector is facile to recover, and for a complex agent system it easier to run the agents, but because it is related to Google's PageRank algorithm (Page et al, 1998). In the PageRank variation, there is a damping factor, suggesting that agents will not always return to a location: i.e., there is a set path length between a and b before the agent gives up. The PageRank algorithm has been recently been applied to axial systems by Jiang (2007), and found to correlate with pedestrian and vehicle movement in urban systems. The damping factor in such a formulation is strongly related to the concept of radius in standard space syntax application.

iii. A similar relationship should hold for the movement expected through an axial line in the stationary distribution and indeed it does appear to hold empirically: theoretically the amount of movement through an axial line should equal the sum of the connectivities of the lines connected to it. This sum is directly proportional to the radius-3 integration (Dalton 2005), which historically has been determined to be the standard movement measure in space syntax.

iv. For more details of the discussion that follows, see Turner (2007).